THE WONDERFUL NUMBER CALLED φ

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What is φ ?

Let us start with any two numbers x and y. We add them together to get the third number x+y.

We now add the second and third numbers to get the fourth number y + x+y = x + 2y, add the third and fourth numbers to get the fifth number x+y + x+2y = 2x + 3y and so on.

Whatever the first two numbers are, the ratio of two successive numbers will tend to 1: 1.618…

This number 1.618… is called φ, the Golden Number.

φ has some remarkable properties.

Its **reciprocal is exactly less by 1,** i.e.

$$1/ \varphi = \varphi-1 \qquad (1)$$

Multiplying althrough by φ, **Its square exceeds φ by one**, i.e.

$$\varphi^2 = \varphi +1, \text{ or, } \varphi^2 - \varphi -1 =0$$

The last relation helps to find its numerical value. Roots of the quadratic equation are :

$$\varphi = ½ [1\pm(1+4)^{½}]$$

Or, ½ [$1\pm\sqrt{5}$], i.e. 1.618 or -0.618.

To see how the ratio converges to 1 : 1.618, start with the numbers 1 and 2

N1	N2	N1+N2	Ratio1:?
1	2	3	1.500
2	3	5	1.6667
3	5	8	1.6000
5	8	13	1.6250
8	13	21	1.6154
13	21	34	1.6190
21	34	55	1.6176
34	55	89	1.6182
55	89	144	1.6180

We can verify the convergence algebraically

Let the numbers be n and nφ.

The next number is $n + n\varphi = n(\varphi+1) = n\varphi^2$

The ratio is $n\varphi : n\varphi^2$ or $1:\varphi$.

The successive ratios are similarly $1:\varphi$.

But if the ratio of the first two numbers is not exactly $1 : \varphi$ ......

Now let the numbers be n and $n(\varphi+e)$, where e, smaller than φ, can be of either sign.

The series would go as :

$n, n(\varphi+e), n(1+\varphi+e), n(1+2\varphi+2e)$ etc.

The ratios are:

$\varphi+e, (1+\varphi+e)/(\varphi+e), (1+2\varphi+2e)/(1+\varphi+e)$ etc.

These can be simplified if $e \ll 1$ and $e/\varphi = f$:

$\varphi(1+f);\ \ \varphi\,[1- f\,(1- f)],\ \varphi\,[1+f/\varphi\,(2/\varphi -1)]$ etc.

The ratios oscillate about φ with gradually diminishing amplitude.

The number is

1.61803398874989484820... (etc.)

There are several ways of arriving at this number. Try the continuous fraction

$$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\dots}}}}$$

If this is φ, then φ-1 = 1/ φ, which is eqn. (1).

- Here is another way:

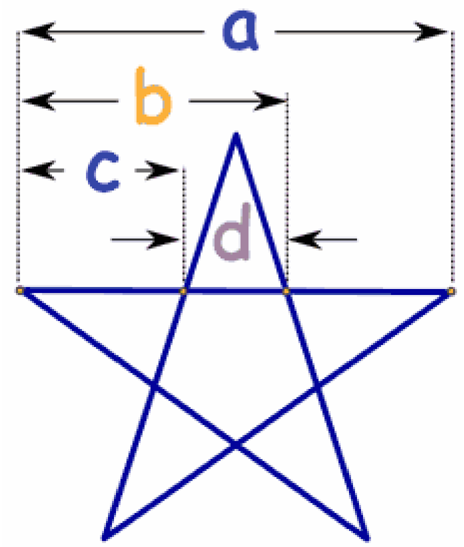
$$\varphi = \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\ldots}}}}$$

Which again yields $\varphi^2 - 1 = \varphi$, which is identical to eqn. (1).

Look at the shape of the structure. Do you think the ratio of its length to breadth pleases your eye ?

Well, the ratio length / breadth is exactly φ.

The Pentagram or the five-pointed cross has φ as the ratio between the legths of its segments.



$$\varphi = a/b$$

$$= b/c$$

$$= c/d$$

The number φ has drawn the fascination of many ancient mathematicians

- Pythagoras and Euclid in ancient Greece
- Leonardo of medieval Italy
- Johannes Kepler of Renaissance period,
- Oxford physicist Roger Penrose.
- Many Biologists, artists, musicians, historians, architects and psychologists have pondered and debated the basis of its ubiquity and appeal.
- The Golden Ratio has inspired thinkers of all disciplines like no other number in the history of mathematics.

The golden ratio has fascinated many.

- The Italian Renaissance mathematician Luca Pacioli wrote a book called "De Divina Proportione" ("The Divine Proportion") in 1509 that discussed and popularized phi.
- The Greek letter Φ was used to denote phi in the 1800s by the American mathematician Mark Barr to represent this number.
- Pacioli used drawings made by Leonardo da Vinci that incorporated phi, and it is possible that da Vinci was the first to call it the (Latin for the "golden section").

- Many wonderful properties have been attributed to phi. Novelist Dan Brown included a long passage in his bestselling book "The Da Vinci Code“, in which the main character discusses how phi represents the ideal of beauty and can be found throughout history. More sober scholars routinely debunk such assertions.
- Phi enthusiasts often mention that certain measurements of the Great Pyramid of Giza, such as the length of its base and/or its height, are in the golden ratio.
- Others claim that the Greeks used phi in designing the Parthenon or in their beautiful statuary.

To conclude, it may be said that the number φ has defeated its competitors “e” and “ π “ in popularity.